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On the Desirability of Nominal GDP Targeting*

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Abstract

This paper evaluates the welfare properties of nominal GDP targeting in the context of a New Keynesian model with both price and wage rigidity. In particular, we compare nominal GDP targeting to inflation and output gap targeting as well as to a conventional Taylor rule. These comparisons are made on the basis of welfare losses relative to a hypothetical equilibrium with flexible prices and wages. Output gap targeting is the most desirable of the rules under consideration, but nominal GDP targeting performs almost as well. Nominal GDP targeting is associated with smaller welfare losses than a Taylor rule and significantly outperforms inflation targeting. Relative to inflation targeting and a Taylor rule, nominal GDP targeting performs best conditional on supply shocks and when wages are sticky relative to prices. Nominal GDP targeting may outperform output gap targeting if the gap is observed with noise, and has more desirable properties related to equilibrium determinacy than does gap targeting.

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1 Introduction

What rule should a central bank follow in the formation of monetary policy? Despite extensive research about this topic, it remains an unsettled question. Although nominal output targeting has recently received attention in the popular press and within policy circles, it has not been scrutinized within the context of the quantitative frameworks commonly used by central banks. The objective of this paper is to study the desirability of nominal GDP targeting within the context of a New Keynesian model with both price and wage rigidity. In particular, we compare the welfare properties of nominal GDP targeting to two other popular targeting rules – inflation and output gap targeting – as well as to a conventional Taylor rule.

Although there is some disagreement on which type of rule central banks should follow, economists agree on several principles in the design of monetary policy. First, rules are preferred to discretion. Rules allow agents to better anchor expectations which improves the inflation-output gap tradeoff. This is true in models of either ad hoc Phillips curves as in Barro and Gordon (1983) or in micro-founded Phillips curves as in Woodford (2003). Second, the policy objectives of central banks should be understandable to the public. As argued by Bernanke and Mishkin (1997), this requires the central bank to be more accountable. Even if monetary policy follows some strict rule, forming expectations is difficult if households do not understand the rule. Third, the central bank faces information constraints which should be taken into account in the formation of monetary policy. Responding to precisely measured variables is superior to responding to imprecisely measured variables or variables that are hypothetical constructs of a model. Finally, a desirable monetary policy rule ought to support a determinate equilibrium.

In the micro-founded welfare loss function from the simplest version of the New Keynesian model, a central bank ought to care about stabilizing both price inflation and the output gap (defined as the gap between the equilibrium level of output and the hypothetical equilibrium level of output which would obtain if prices were flexible). Inflation and output gap targeting are therefore easily motivated as potentially desirable policy rules. In a version of the model where only prices are sticky, the so-called "Divine Coincidence" (Blanchard and Gali (2007)) holds, and either targeting rule fully implements the flexible price allocation. In other words, stabilizing inflation also stabilizes the output gap, and vice-versa. Given that the output gap is a hypothetical model-based construct, whereas inflation is easily observed at high frequencies, inflation targeting is therefore often touted as a highly desirable and easily implemented policy rule.

Inflation targeting may be less desirable in versions of the model in which the Divine Coincidence does not hold. Erceg et al. (2000) consider a version of the model in which

both prices and nominal wages are sticky, the combination of which breaks the equivalence between inflation and gap stabilization and renders it impossible for the central bank to fully implement the flexible price and wage allocation. For plausible parameterizations of the parameters governing price and wage stickiness, they find that inflation targeting tends to perform poorly from a welfare perspective. Output gap targeting, in contrast, does very well, coming close to implementing the flexible price and wage allocation.

Although nominal GDP targeting has recently gained attention as potential policy rule in the popular press and within policy circles, it has received relatively little attention within the context of the type of models in widespread use at central banks and among academics. Whereas inflation targeting focuses on nominal variables and gap targeting on real variables, nominal GDP targeting simultaneously targets both nominal and real variables. And unlike output gap targeting, it does not require the central bank to observe the hypothetical flexible price and wage level of output.

In Section 2 we study the merits of nominal GDP targeting relative to other policy rules using a standard parameterization of the sticky price and wage New Keynesian model developed by Erceg et al. (2000). We evaluate different rules on the basis of the average welfare loss relative to a hypothetical flexible price and wage allocation. Like Erceg et al. (2000), we find that output gap targeting does well, producing very small welfare losses that come close to implementing the flexible price and wage allocation. Nominal GDP targeting does almost as well as gap targeting. It is associated with smaller welfare losses than a conventionally parameterized Taylor rule and significantly outperforms inflation targeting. Nominal GDP targeting performs best in a relative sense when wages are sticky relative to prices and conditional on supply shocks. These results are consistent with the intuition laid out by Sumner (2014) in a textbook aggregate supply / aggregate demand model.

We consider a more elaborate medium scale version of the model in Sections 3 and 4. In addition to price and wage stickiness, the model features investment and capital accumulation, several sources of real inertia, and a number of different shocks. In Section 4, the parameters of the model are estimated using Bayesian methods to fit recent US data. The results of the medium scale model echo those from the small scale model. Output gap targeting produces the lowest welfare losses relative to the hypothetical flexible price and wage allocation. Nominal GDP targeting does almost as well, significantly bettering both an estimated Taylor rule as well inflation targeting. As in the small scale model, nominal GDP targeting performs best when wages are sticky relative to prices and conditional on supply shocks, although the relative desirability of nominal GDP targeting conditional on supply and demand shocks is not as stark as in the small scale model.

Though we find output gap targeting to be the best performing of the different policy

rules under consideration, there are some reasons to be weary of actually implementing gap targeting as a policy rule. These reasons relate back to some of the basic principles of desirable monetary policy which are highlighted at the beginning of the Introduction. First, as a hypothetical model construct, gap targeting may be difficult to successfully communicate to the public, even if the central bank can observe the flexible price and wage level of output with precision. Second, it may be difficult for the central bank to observe the flexible price and wage level of output, particularly in real time. This point has been made by Orphanides (2001), Orphanides and van Norden (2002), and Orphanides (2003). Third, gap targeting may result in equilibrium indeterminacy. This point has been made in the context of sticky price New Keynesian models with positive trend inflation in Hornstein and Wolman (2005), Ascari and Ropele (2009), and Coibion and Gorodnichenko (2011).

We consider in Section 4 a couple of different exercises to address some of these points in the context of the estimated medium scale model. In one, we assume that the central bank observes the flexible price and wage level of output with noise, and compute the amount of noise which would equate the welfare losses associated with targeting the mis-measured gap and targeting nominal GDP. We find that if the noise in the observed flexible price and wage level of output is more than about one-third the volatility of the actual flexible price and wage level of output, then nominal GDP targeting produces a lower welfare loss than does gap targeting. In a second exercise, we suppose that the central bank adjusts its perceived flexible price and wage level of output to the actual flexible price and wage level of output slowly. If the central bank adjusts slowly enough, then gap targeting can result in significant welfare losses relative to nominal GDP targeting. Third, we investigate the determinacy properties of gap targeting when trend inflation is positive. We find that if trend inflation is greater than about 0.2 percent annually, then gap targeting results in equilibrium indeterminacy. Because in the long run the level of output is independent of monetary policy, nominal GDP targeting is equivalent to a price level target in the long run, and therefore supports a determinate equilibrium for any level of trend inflation.

Our paper is related to a large literature on optimal monetary policy. The relative merits of nominal GDP targeting versus inflation targeting have been recently revived by Billi (2014) and Woodford (2012), who discuss the rules within the context of the zero lower bound. Cecchetti (1995) and Hall and Mankiw (1994) show in counterfactual simulations that nominal GDP targeting would lower the volatility of both real and nominal variables. Neither of these latter two papers have a structural model to conduct welfare analysis. Mitra (2003) studies nominal GDP targeting in a small scale New Keynesian model with adaptive learning. Jensen (2002) compares inflation and nominal GDP targeting in a linearized New Keynesian model with price stickiness and Kim and Henderson (2005) compare the two rules in a model

of wage and price stickiness. While both of these papers include structural models similar to our own, Jensen (2002) does not include wage stickiness and Kim and Henderson (2005) uses one period ahead price and wage contracts rather than the more common staggered price and wage contracts. Moreover, Mitra (2003), Jensen (2002), and Kim and Henderson (2005) do not analyze nominal GDP targeting in a medium scale model like Smets and Wouters (2007) or Justiniano et al. (2010) which are extensively used in central banks. Schmitt-Grohe and Uribe (2007) study optimal policy rule coefficients for a Taylor rule in a sticky price New Keynesian model with capital. They find that it is optimal to respond strongly to inflation and not at all to output, though they do not allow the central bank to target the output gap and their model does not feature wage rigidity. Sims (2013) studies the relative merits of a Taylor rule responding to the output gap or output growth.

2 The Basic New Keynesian Model

We begin by studying a textbook New Keynesian model featuring both wage and price rigidity along the lines of Erceg et al. (2000). This model is a special case of the medium scale model we study in the next section. In the subsections below we briefly describe the problems of each type of agent in the model and discuss equilibrium and aggregation. We then use the model to develop some intuition for the relative benefits of different monetary policy rules. The full set of conditions characterizing the equilibrium are presented in Appendix A.

2.1 Households

There exist a continuum of households indexed by $h \in [0,1]$. These households are monopoly suppliers of differentiated labor, $N_t(h)$. There exists a labor aggregating firm which bundles differentiated labor input into an aggregate labor input, N_t, which is sold to firms at real wage w_t; $w_t(h)$ denotes the real wage paid to household h. The technology which bundles labor intput is given by:

$$N_t = \left(\int_0^1 N_t(h)^{\frac{\epsilon_w - 1}{\epsilon_w}} \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad \epsilon_w > 1. \tag{1}$$

The parameter ϵ_w measures the degree of substitutability among different types of labor. Profit maximization by the labor aggregating firm gives rise to a downward sloping demand curve for each variety of labor and an aggregate real wage index:

$$N_t(h) = \left(\frac{w_t(h)}{w_t} \right)^{-\epsilon_w} N_t \tag{2}$$

$$w_t^{1-\epsilon_w} = \int_0^1 w_t(h)^{1-\epsilon_w} dh. \tag{3}$$

Households are not freely able to adjust their wage each period. In particular, each period there is a $1-\theta_w$ probability, $\theta_w \in [0,1)$, that a household can adjust its wage. This probability is independent of when a household last adjusted its wage. As in Erceg et al. (2000), we assume there exists state-contingent securities which insure households against idiosyncratic wage risk. We also assume that preferences are separable in consumption and labor. Combined with perfect insurance, this means that households will make identical non-labor market choices. We therefore suppress formal dependence on h in writing the problem of a particular household with the exception of labor market variables.

The problem of a particular household can be written:

$$\max_{C_t, B_t, w_t(h), N_t(h)} \quad E_0 \sum_{t=0}^{\infty} \beta^t \nu_t \left\{ \ln C_t - \psi \frac{N_t(h)^{1+\eta}}{1+\eta} \right\}$$

subject to

$$C_t + \frac{B_t}{P_t} \leq w_t(h) N_t(h) + \Pi_t + (1+i_{t-1}) \frac{B_{t-1}}{P_t} \tag{4}$$

$$N_t(h) \geq \left(\frac{w_t(h)}{w_t} \right)^{-\epsilon_w} N_t \tag{5}$$

$$w_t(h) = \begin{cases} w_t^{\#}(h) & \text{if } w_t(h) \text{ chosen optimally} \\ (1+\pi_t)^{-1} w_{t-1}(h) & \text{otherwise} \end{cases}. \tag{6}$$

The discount factor is given by $\beta \in (0,1)$, ψ is a scaling parameter on the disutility from labor, and η represents the inverse of the Frisch elasticity of labor supply. The exogenous variable ν_t is a preference shock common to all households. Constraint (4) is a standard flow budget constraint. A household enters the period with a stock of nominal bonds, B_{t-1}, and can choose a new stock of bonds, B_t, which pay out nominal interest rate i_t in period $t+1$. Consumption is denoted by C_t and the price of goods is P_t. Real profit distributed from firms is Π_t. Constraint (5) requires that household labor supply meets demand. Wage-setting is described by (6). With probability $1-\theta_w$ the household chooses a new real wage, denoted by $w_t^{\#}(h)$. With probability θ_w the household is unable to adjust its nominal wage, so the real wage it charges in period t is its period $t-1$ real wage divided by the gross inflation rate between $t-1$ and t, where $1+\pi_t = P_t/P_{t-1}$. Optimization gives rise to a standard Euler equation for bonds that is the same across all households. It is straightforward to show that all households given the opportunity will adjust to a common wage, $w_t^{\#}$.

2.2 Production

Production takes place in two phases. There exist a continuum of producers of differentiated output indexed by $j \in [0,1]$, $Y_t(j)$. Differentiated output is transformed into final output, Y_t, by a competitive firm using the following technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \quad \epsilon_p > 1. \tag{7}$$

The parameter ϵ_p measures the degree of substitutability among differentiated output. The price of the final output is P_t and the prices of differentiated output are denoted by $P_t(j)$. Profit maximization by the competitive firm gives rise to demand for each differentiated output and an aggregate price index:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t \tag{8}$$

$$P_t^{1-\epsilon_p} = \int_0^1 P_t(j)^{1-\epsilon_p}. \tag{9}$$

The production function for producer j is given by:

$$Y_t(j) = A_t N_t(j). \tag{10}$$

The exogenous variable A_t is a productivity shock common across all producers of differentiated output. These firms are not freely able to adjust their price in a given period in an analogous way to household wage-setting. In particular, each period there is a $1 - \theta_p$ probability, $\theta_p \in [0,1)$, that a firm can adjust its price, which we denote by $P_t^{\#}(j)$. Otherwise it charges its most recently chosen price:

$$P_t(j) = \begin{cases} P_t^{\#}(j) & \text{if } P_t(j) \text{ chosen optimally} \\ P_{t-1}(j) & \text{otherwise} \end{cases}. \tag{11}$$

Regardless of whether a firm can adjust its price, it will choose labor input to minimize cost, subject to the constraint of producing enough to meet demand, given by (8). Solving the cost minimization problem reveals that all firms face the same real marginal cost, given by $mc_t = w_t / A_t$. Firms given the opportunity to adjust their price will do so to maximize the expected presented discounted value of profit returned to households. It is straightforward to show that all updating firms will choose a common reset price, $P_t^{\#}$.

2.3 Exogenous Processes

There are two exogenous variables in the model, the productivity shock, A_t, and the preference shock, ν_t. We assume that these both follow stationary AR(1) processes with non-stochastic means normalized to unity:

$$\ln A_t = \rho_A \ln A_{t-1} + \sigma_A \varepsilon_{A,t} \tag{12}$$

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \sigma_\nu \varepsilon_{\nu,t}. \tag{13}$$

The autoregressive parameters, ρ_A and ρ_ν, lie between zero and one. The innovations, $\varepsilon_{A,t}$ and $\varepsilon_{\nu,t}$, are drawn from standard normal distributions. These innovations are scaled by σ_A and σ_ν, which measure the standard deviations of the innovations.

2.4 Market-Clearing and Aggregation

Market-clearing requires that bond-holding is zero at all times, $B_t = 0$, and that the sum of labor demanded by producers of differentiated output equals total labor supplied by the labor aggregating firm. Integrating the flow budget constraints of households along with the definition of firm profits gives the aggregate resource constraint $Y_t = C_t$. The price and real wage indexes can be written without reference to household or firm subscripts as:

$$P_t^{1-\epsilon_p} = (1-\theta_p) P_t^{\#,1-\epsilon_p} + \theta_p P_{t-1}^{1-\epsilon_p} \tag{14}$$

$$w_t^{1-\epsilon_w} = (1-\theta_w) w_t^{\#,1-\epsilon_w} + \theta_w (1+\pi_t)^{\epsilon_w - 1} w_{t-1}^{1-\epsilon_w}. \tag{15}$$

Integrating over the demand curves for differentiated output, (8), gives rise to an aggregate production function:

$$Y_t = \frac{A_t N_t}{v_t^p} \tag{16}$$

where the variable v_t^p is a measure of price dispersion given by:

$$v_t^p = \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_p} dj. \tag{17}$$

We define the hypothetical construct of the flexible price level of output (or sometimes "natural rate") as the level of output which would obtain in the absence of price and wage stickiness. We denote this by Y_t^f, and can solve for it as the equilibrium level of output when $\theta_p = \theta_w = 0$. We then define the output gap, X_t, as the ratio of the level of output to the

flexible price level, $X_t = Y_t/Y_t^f$.

Aggregate welfare is defined as the sum of the presented discounted value of flow utility across all households. This can be written recursively in terms of aggregate variables alone as:

$$\mathbb{W}_t = \nu_t \left[\ln C_t - \psi v_t^w \frac{N_t^{1+\eta}}{1+\eta} \right] + \beta E_t \mathbb{W}_{t+1}. \tag{18}$$

Recall that N_t is aggregate labor supplied by the labor aggregating firm. This is only equal to aggregate labor supply in the special case that all households charge identical wages. In the more general case, there is a wedge between aggregate labor supply and labor used in production due to wage dispersion. This is captured by the variable v_t^w, which can be written:

$$v_t^w = \int_0^1 \left(\frac{w_t(h)}{w_t} \right)^{-\epsilon_w(1+\eta)} dh. \tag{19}$$

2.5 Monetary Policy

Before fully characterizing the equilibrium it remains to specify the conduct of monetary policy. In addition to a nominal GDP targeting rule, we also consider inflation and output gap targeting rules. Each of these three targeting rules can be considered restricted cases of a generalized Taylor (1993) type instrument rule, which is often used in quantitative work to model the behavior of monetary policy.

The primary objective of the paper is to evaluate the performance of a nominal GDP targeting rule vis-á-vis other popular targeting rules. A nominal GDP targeting rule can be written as

$$P_t Y_t = (PY)^*. \tag{20}$$

In this rule the nominal interest rate adjusts so that nominal GDP, $P_t Y_t$, equals some exogenous and constant target, $(PY)^*$.[1] In the long run this rule is equivalent to a price level target since the long run level of real GDP is independent of the monetary policy rule. In the short run, commitment to this rule implies that the growth rate of nominal GDP is zero, or that $(1+\pi_t)\frac{Y_t}{Y_{t-1}} = 1$ each period.

The second rule we consider is a strict inflation targeting rule:

$$\pi_t = 0. \tag{21}$$

Under an inflation targeting rule the central bank adjusts the nominal interest such that

[1] Here we follow convention in assuming no trend growth in output or trend inflation. It is straightforward to modify the nominal GDP targeting rule to account for these features.

inflation hits its target each period (implicitly we have normalized the target inflation rate to zero, but could easily extend this to non-zero targets).

The final targeting rule we consider is an output gap targeting rule:

$$X_t = 1. \tag{22}$$

In this rule the nominal interest rate is adjusted in such a way that the equilibrium level of output always equals its flexible price level.

In a version of the model with flexible wages the so-called "Divine Coincidence" would hold (Blanchard and Gali (2007)), and in equilibrium inflation and gap targeting would be equivalent to one another, and would both implement the flexible price equilibrium. When prices and wages are simultaneously sticky this equivalence does not hold. Gap targeting requires that the central bank know the flexible price level of output, which is not directly observable, neither in real time nor ex-post. A potential advantage of inflation targeting is that inflation is observed at high frequencies and does not require a central a bank to know anything about the underlying model. Whereas gap targeting focuses on a real variable and inflation targeting focuses solely on prices, nominal GDP targeting implicitly targets both nominal and real variables. It also only requires a central bank to observe endogenous variables, not hypothetical model constructs like the flexible price level of output.

Each of the three targeting rules can be understood to be special cases of a generalized Taylor rule of the following form:

$$\ln(1+i_t) = (1-\rho_i)\ln(1+i^*) + \rho_i \ln(1+i_{t-1}) + \phi_\pi \ln(1+\pi_t) + \phi_x \ln X_t + \phi_y \ln\left(\frac{Y_t}{Y_{t-1}}\right). \tag{23}$$

Here $\rho_i \in [0,1)$ is a smoothing parameter and ϕ_π, ϕ_x, and ϕ_y are non-negative coefficients on inflation, the output gap, and output growth, respectively. In writing this rule we have maintained an implicit assumption of zero trend inflation and no trend growth. Whereas the three rules discussed above are targeting rules, targeting the values of endogenous variables and adjusting the nominal rate to hit those targets, the Taylor rule is an instrument rule, directly specifying a process for the nominal interest rate as function of endogenous variables. Taylor rules of this form seem to provide an accurate account of observed monetary policy in the last several decades and tend to have good normative properties. Each of the three targeting rules can be understood to be special cases of the Taylor rule. The Taylor rule would be isomorphic to an inflation targeting rule when $\phi_\pi \to \infty$ and $\rho_i = \phi_x = \phi_y$ and equivalent to a gap targeting rule when $\phi_x \to \infty$ and $\rho_i = \phi_\pi = \phi_y$. Because commitment to a nominal GDP targeting rule implies targeting the growth rate of nominal GDP each period, it is equivalent

to the Taylor rule when $\phi_\pi = \phi_y \to \infty$ and $\rho_i = \phi_x = 0$.[2]

2.6 Quantitative Analysis

We adopt a standard parameterization of the model's parameters; in the next section with a more empirically plausible model with capital accumulation we estimate them. The values of the parameters are listed in Table 1.

Table 1: Value of Parameters

Parameter	Description	Value
β	Discount rate	0.995
η	Inverse Frisch elasticity	1
ψ	Labor disutility	$N^* = 1/3$
ϵ_w	Elasticity of substitution – Labor	10
ϵ_p	Elasticity of substitution – Goods	10
θ_w	Wage stickiness	0–0.75
θ_p	Price stickiness	0–0.75
σ_A	Standard deviation – Productivity	0.006
σ_ν	Standard deviation – Preference	0.020
ρ_A	Persistence – Productivity	0.97
ρ_ν	Persistence – Preference	0.70

Notes: The table shows the values of the parameters uses in the quantitative analysis of the small scale model laid out in this section.

The discount factor is set to $\beta = 0.995$, which implies an annualized risk-free interest rate of two percent. The Frisch labor supply elasticity is set to unity, implying $\eta = 1$. The elasticities of substitution for goods and labor, ϵ_p and ϵ_w, are both set to 10, implying steady state price and wage markups of a little more than ten percent. The scaling parameter on the disutility from labor, ψ, is set so that steady state labor hours equal 1/3. Our baseline parameterization of the price and wage stickiness parameters is $\theta_w = \theta_p = 0.75$, which implies that wages and prices both adjust once a year on average. We consider various different values of these parameters in the quantitative work below. The autoregressive parameter in the productivity process is set to $\rho_A = 0.97$ and the autoregressive parameter for the preference shock is $\rho_\nu = 0.70$. The standard deviations of the innovations to productivity and

[2]One could also map the Taylor rule into targeting rules that are not strict in the sense of featuring large but nevertheless finite coefficients on the target variables in the Taylor rule. In particular, an inflation targeting Taylor rule might feature a "large" value of ϕ_π (say, 10) and values of 0 for the other parameters, and similarly for a gap target and the coefficient on ϕ_x and a nominal GDP target with large and equal coefficients on ϕ_π and ϕ_y. We have experimented with these specifications and obtain similar results to the strict targeting rules considered.

preferences are $\sigma_A = 0.006$ and $\sigma_\nu = 0.02$, respectively. When monetary policy is governed by a conventionally parameterized version of the Taylor rule with $\rho_i = 0.7$, $\phi_\pi = 0.45$, $\phi_y = 0.0375$, and $\phi_x = 0$,[3] this parameterization generates a standard deviation of log output of 0.036, which is very close to the observed volatility of linearly detrended log GDP in postwar US data. Furthermore, the productivity and preference shocks each account for 50 percent of the unconditional variance of log output. In other words, our parameterization implies that supply and demand shocks are equally important in driving fluctuations in output.

We evaluate different policy rules by computing the unconditional mean of welfare for a particular policy rule and comparing that to the unconditional mean of welfare in a hypothetical economy where prices and wages are both flexible, e.g. $\theta_p = \theta_w = 0$. We compute these unconditional means by solving the model using a second order approximation of the equilibrium conditions about the non-stochastic steady state. We then calculate a compensating variation, computing the percent of consumption each period which would make a household indifferent between the flexible price and wage economy and the sticky price and wage economy. Formally, the compensating variation is given by:

$$\lambda = 100\left[\exp(\mathbb{E}\,\mathbb{W}_{flex} - \mathbb{E}\,\mathbb{W}_0) - 1\right]$$

where $\mathbb{E}\,\mathbb{W}_{flex}$ is the expected present discounted value of utility under flexible prices and wages and $\mathbb{E}\,\mathbb{W}_0$ is the expected discounted value of utility in the sticky price and wage economy with a given monetary policy regime. We can interpret this compensating variation as a welfare loss from price and wage rigidity. More desirable monetary policy regimes therefore coincide with lower values of the compensating variation.

Table 2 present compensating variations for the three different targeting rules for different combinations of price and wage stickiness parameters. For point of comparison we also show the compensating variation for the Taylor rule parameterized as described above. Focus first on the case where the price and wage rigidity parameters are both 0.75. The most desirable policy regime is the output gap targeting rule, which produces a compensating variation of only 0.02 percent of consumption. Nominal GDP targeting does almost as well, generating a welfare loss of only 0.03 percent of consumption. Inflation targeting performs very poorly, with a welfare loss of nearly 20 percent of consumption. The Taylor rule performs fairly well, albeit substantially worse than either nominal GDP or output gap targeting, with a compensating variation of 0.3 percent.

We now turn attention to different combinations of price and wage stickiness parameters. The output gap targeting rule at least weakly dominates the other two targeting rules as well

[3]Note that our rule is not written as one of partial adjustment. The long run response of the interest rate to inflation and output growth is $\phi_\pi/(1-\rho_i)$ and $\phi_y/(1-\rho_i)$, or 1.5 and 0.125, respectively.

as the Taylor rule for all combinations of price and wage rigidity. The output gap targeting rule implements the flexible price allocation (i.e. the compensating variation is zero) if either prices or wages are flexible. Inflation targeting tends to perform very poorly except in the extreme case where wages are flexible, when strict inflation targeting also fully implements the flexible price allocation. The compensating variation associated with the Taylor rule is roughly the same at all combinations of price and wage rigidity under consideration. Nominal GDP targeting tends to perform very well, yielding compensating variations less than 0.04 percent of consumption for all combinations of price and wage stickiness parameters under consideration.

Table 2: Consumption Equivalent Welfare Losses from Different Policy Rules

	$\theta_p = 0.75$ $\theta_w = 0.75$	$\theta_p = 0.00$ $\theta_w = 0.75$	$\theta_p = 0.50$ $\theta_w = 0.75$	$\theta_p = 0.75$ $\theta_w = 0.50$	$\theta_p = 0.75$ $\theta_w = 0.00$
NGDP	0.0314	0.0000	0.0127	0.0352	0.0398
Inflation	20.3694	20.3694	20.3694	0.8618	0.0000
Output Gap	0.0190	0.0000	0.0090	0.0166	0.0000
Taylor Rule	0.2880	0.2908	0.2902	0.2506	0.1601

Notes: The table contains the compensating variations of three different targeting rules as well as a Taylor rule. The parameterization of the model is as described in Table 1. The first three rows consider strict targeting rules. The row labeled "Taylor rule" considers the Taylor rule, (23), parameterized with $\rho_i = 0.7$, $\phi_\pi = 0.45$, $\phi_x = 0$, and $\phi_y = 0.0375$.

That inflation targeting performs poorly, and output gap targeting does very well, when both prices and wages are rigid has been well-known since Erceg et al. (2000). The novel result here is that nominal GDP targeting seems to have desirable properties. Nominal GDP targeting is associated with substantially lower compensating variations than the Taylor rule, and significantly outperforms inflation targeting except in the case where wages are flexible. While nominal GDP targeting is always weakly worse than output gap targeting, the differences in the compensating variations associated with these two targeting rules are very small. From the Table it appears as though nominal GDP targeting is relatively better the stickier are wages relative to prices – when wages are rigid and prices are flexible, nominal GDP targeting is equivalent to gap targeting, while when wages are flexible and prices rigid, inflation targeting outperforms nominal GDP targeting.

Figure 1 shows the loci of wage (horizontal axis) and price (vertical axis) rigidity parameters where nominal GDP targeting and inflation targeting generate the same compensating variation. The area shaded green shows parameter combinations where nominal GDP targeting strictly dominates inflation targeting. For wage stickiness parameters in excess

of 0.3 nominal GDP targeting strictly dominates inflation targeting for any value of the parameter governing price rigidity. Furthermore, at low levels of wage stickiness inflation targeting only dominates nominal GDP targeting if prices are very rigid.

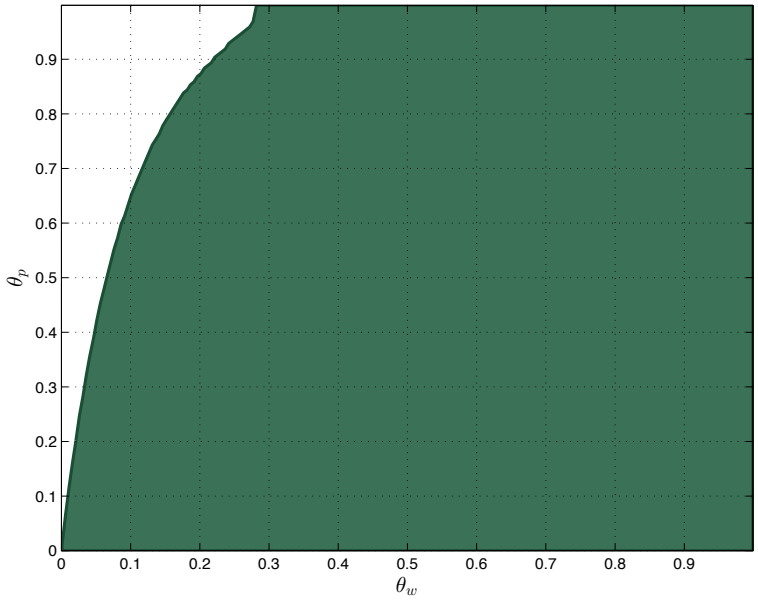

Figure 1: Nominal GDP vs. Inflation Targeting

Notes: This figure traces out the loci of wage stickiness parameters, (θ_w, θ_p) for which nominal GDP targeting and inflation targeting yield the same compensating variation. The area shaded green depicts combinations of (θ_w, θ_p) for which nominal GDP targeting strictly dominates inflation targeting.

We next consider the role of the two different shocks in the model in driving the relative performance of the different monetary policy regimes. Tables 3 and 4 repeat the exercises in Table 2 conditioning on only the productivity or the preference shock, respectively. For this exercise, we set the standard deviation of one of the shocks to zero, and re-parameterize the standard deviation of the remaining shock to generate the same volatility of log output as in the baseline model when policy is characterized by a Taylor rule.[4]

The compensating variations for the different policy regimes when there are only productivity shocks are qualitatively similar to the compensating variations when both shocks are included in the model. Output gap targeting always weakly dominates the other policies and inflation targeting does poorly unless wages are flexible. Nominal GDP targeting is significantly more desirable than inflation targeting unless wages are flexible, and performs better than the Taylor rule for most parameter configurations. Inflation targeting performs worse conditional on productivity shocks relative to the case where both shocks are in the

[4]When conditioning on the productivity shock, this implies a value of $\sigma_A = 0.0085$ and $\sigma_\nu = 0$. When conditioning on the preference shock, $\sigma_A = 0$ and $\sigma_\nu = 0.0285$.

model. When there are only preference shocks, each of the three targeting regimes implements the flexible price allocation for any combination of price and wage rigidity parameters, while the Taylor rule performs similarly to when both shocks are in the model. This equivalence of the three targeting regimes obtains because of the absence of capital in the model and our assumptions about preferences. Given that preferences are separable in consumption and labor and that consumption must equal output, in the flexible price allocation neither output nor real wages change in response to a preference shock.[5] This has the implication that stabilizing prices (inflation targeting), real activity (gap targeting), or a mix of both (nominal GDP targeting) are equivalent in equilibrium.

Table 3: Consumption Equivalent Welfare Losses from Different Policy Rules, Only Productivity Shocks

	$\theta_p = 0.75$ $\theta_w = 0.75$	$\theta_p = 0.00$ $\theta_w = 0.75$	$\theta_p = 0.50$ $\theta_w = 0.75$	$\theta_p = 0.75$ $\theta_w = 0.50$	$\theta_p = 0.75$ $\theta_w = 0.00$
NGDP	0.0629	0.0000	0.0254	0.0707	0.0800
Inflation	45.0747	45.0747	45.0747	1.7372	0.0000
Output gap	0.0382	0.0000	0.0180	0.0333	0.0000
Taylor rule	0.2740	0.2767	0.2767	0.0937	0.0178

Notes: The table contains the compensating variations of the policies described in the text in a specification of the model in which there are only productivity shocks. In particular, the standard deviation of the productivity shock is chosen to produce the same volatility under a Taylor rule as in the version of the model with both shocks, while the standard deviation of the preference shock is set to zero. The standard deviation of the productivity shock is $\sigma_A = 0.0085$.

The results in Tables 3 and 4 suggest that the choice of monetary regime is far more relevant for welfare conditional on supply shocks (i.e. the productivity shock) than demand shocks (i.e. the preference shock). This confirms the simple aggregate demand - supply intuition in Sumner (2014). Our baseline parameterization assigns the productivity and preference shocks equal weight in accounting for output fluctuations, and therefore takes a relatively agnostic stand on the relative importance of supply and demand shocks. In the next section we consider an empirically realistic medium scale model and estimate the relative importance of several different kinds of shocks. In the simple model of this section, the demarcation between supply and demand shocks is clear, because the preference shock

[5] A simple way to see this is to note that the price and wage markups are fixed when both prices and wages are flexible. Given our assumptions on preferences, this means that consumption and labor must co-move negatively absent a change in A_t. But since $C_t = Y_t = A_t N_t$, consumption and labor cannot co-move negatively absent a change in A_t. Hence, in the flexible price allocation consumption, hours, output, and the real wage do not react to a preference shock.

does not impact the flexible price level of output. But when capital and other features are added to the model, the distinction between "supply" and "demand" can become blurry. While the basic intuition about the choice of policy regime conditional on supply and demand shocks will indeed carry over largely intact, some caution is in order when extrapolating from the simple model to the more realistic one in the next section.

Table 4: Consumption Equivalent Welfare Losses from Different Policy Rules, Only Preference Shocks

	$\theta_p = 0.75$ $\theta_w = 0.75$	$\theta_p = 0.00$ $\theta_w = 0.75$	$\theta_p = 0.50$ $\theta_w = 0.75$	$\theta_p = 0.75$ $\theta_w = 0.50$	$\theta_p = 0.75$ $\theta_w = 0.00$
NGDP	0.0000	0.0000	0.0000	0.0000	0.0000
Inflation	0.0000	0.0000	0.0000	0.0000	0.0000
Output gap	0.0000	0.0000	0.0000	0.0000	0.0000
Taylor rule	0.3094	0.3105	0.3094	0.4144	0.3078

Notes: The table contains the compensating variations of the policies described in the text in a version of the model in which there are only preference shocks. In particular, the standard deviation of the preference shock is chosen to produce the same volatility under a Taylor rule as in the version of the model with both shocks, while the standard deviation of the productivity shock is set to zero. The standard deviation of the preference shock is $\sigma_\nu = 0.0285$.

3 Medium Scale Model

While the previous section allows one to understand some of the intuition for the welfare properties of nominal GDP targeting relative to inflation targeting and a Taylor rule, it did so within the context of a very simplified framework. In this section, we consider a medium scale version of the model. The simple economy of the previous section is a special case of the more realistic model in this section. In addition to price and wage rigidity, the model allows for capital accumulation, habit formation in consumption, variable capital utilization, wage and price indexation to lagged inflation, and several more stochastic shocks. Such a model has been shown to capture the dynamic effects of monetary policy and the most salient business cycle facts.[6] Where different from the simpler model of Section 2, in the subsections below we lay out the details of the medium scale model. The full set of conditions characterizing the equilibrium is presented in Appendix B.

[6]See Christiano et al. (2005) as an example of the former and Smets and Wouters (2007) for the latter.

3.1 Households

The basic structure of the household side of the model is very similar to the simpler model. There again exist a continuum of households indexed by $h \in [0,1]$. The labor aggregating firm is the same, giving rise to the same downward-sloping demand for each household's labor, (2), and the same aggregate real wage index, (3). The medium scale model departs from the simpler model in allowing for internal habit formation in consumption, capital accumulation and capital utilization, and indexation of wages to lagged inflation. The problem of a particular household is:

$$\max_{C_t,B_t,u_t,I_t,K_{t+1},w_t(h),N_t(h)} \quad E_0 \sum_{t=0}^{\infty} \beta^t \nu_t \left\{ \ln\left(C_t - bC_{t-1}\right) - \psi \frac{N_t(h)^{1+\eta}}{1+\eta} \right\}$$

subject to

$$C_t + I_t + \frac{B_t}{P_t} + \left[\gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2\right] K_t \leq w_t(h) N_t(h) + R_t u_t K_t + \Pi_t + T_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} \quad (24)$$

$$K_{t+1} = Z_t \left[1 - \frac{\tau}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1-\delta) K_t \quad (25)$$

$$N_t(h) \geq \left(\frac{w_t(h)}{w_t}\right)^{-\epsilon_w} N_t \quad (26)$$

$$w_t(h) = \begin{cases} w_t^{\#}(h) & \text{if } w_t(h) \text{ chosen optimally} \\ (1+\pi_t)^{-1}(1+\pi_{t-1})^{\zeta_w} w_{t-1}(h) & \text{otherwise} \end{cases} . \quad (27)$$

Relative to the simpler model, the preference specification is identical with the exception of the inclusion of internal habit formation, which is governed by the parameter $b \in [0,1)$. In the flow budget constraint, (24), I_t denotes investment, K_t physical capital, u_t capital utilization, R_t the rental rate on capital services (where capital services is understood to represent the product of utilization and physical capital), and T_t is a lump sum tax paid to a government. There is a convex resource cost of capital utilization given by $\gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2$. This cost is measured in units of physical capital. The law of motion for physical capital is given by (25). Z_t is an exogenous shock to the marginal efficiency of investment along the lines of Justiniano et al. (2010) and $\tau \geq 0$ is an investment adjustment cost in the form proposed by Christiano et al. (2005). The depreciation rate on physical capital is given by $\delta \in (0,1)$. A household is required to supply as much labor as is demanded at its wage, just as in the simpler model. The wage-setting process is virtually the same as in the simpler model. With probability $1 - \theta_w$ a household can adjust to a new optimal wage, which we denote in real terms as $w_t^{\#}(h)$. Otherwise a household must charge its most recently chosen nominal

wage. Differently than the simple model, here we permit partial indexation of nominal wages to lagged inflation as measured by the parameter $\zeta_w \in [0,1]$. It is again the case that all updating households will adjust to the same reset real wage, which we denote by $w_t^{\#}$.

3.2 Production

The production process again takes place in two phases. The final output aggregator, demand curve for each differentiated output, and expression for the aggregate price level are the same as in the simpler model, given by equations (7), (8), and (9), respectively.

The production function of firms who produce differentiated output is given by:

$$Y_t(j) = \max\left\{A_t \widehat{K}_t(j)^{\alpha} N_t(j)^{1-\alpha} - F, 0\right\}. \tag{28}$$

Here $\widehat{K}_t$ is capital services leased from households. The parameter α lies between 0 and 1. There is also a fixed cost of production given by $F \geq 0$. Cost-minimization by firms reveals that all firms face the same real marginal cost and hire capital services and labor in the same ratio. As in the simpler model, firms face a probability of $1-\theta_p$ that they can adjust their price in a given period. Differently than the simpler model, we permit indexation of non-updated prices to lagged inflation, governed by the parameter $\zeta_p \in [0,1]$. A firm's price in any period therefore satisfies:

$$P_t(j) = \begin{cases} P_t^{\#}(j) & \text{if } P_t(j) \text{ chosen optimally} \\ (1+\pi_{t-1})^{\zeta_p} P_{t-1}(j) & \text{otherwise} \end{cases}. \tag{29}$$

Updating firms will choose their prices to maximize the present discounted value of flow profits, where discounting is by the stochastic discount factor of households.[7] It is again straightforward to show that all updating firms choose an identical price, denoted by $P_t^{\#}$.

3.3 Fiscal Policy

In the medium scale model we allow for government spending. Government spending is assumed to be exogenous and obeys the following stationary stochastic process:

$$\ln G_t = (1-\rho_G)\ln G^* + \rho_G \ln G_{t-1} + \sigma_G \varepsilon_{G,t} \tag{30}$$

[7] Though there is heterogeneity among households, because of separability and perfect insurance the marginal utility of income is identical across households, so it is safe to talk about one stochastic discount factor in spite of the heterogeneity among households.

Here G^* is the steady state level of government spending, the parameter ρ_G lies between zero and one, and the innovation $\varepsilon_{G,t}$ is drawn from a standard normal distribution. The innovation is scaled by the standard deviation parameter σ_G. We assume that the only source of government revenue is lump sum taxes, T_t. This means that it is innocuous to assume that the government balances its budget each period, with $G_t = T_t$.

3.4 Exogenous Processes

The exogenous processes for productivity, A_t, and the preference shock, ν_t, are the same as in Section 2. We assume that the investment shock, Z_t, also obeys a stationary stochastic process with steady state value normalized to unity:

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \sigma_Z \varepsilon_{Z,t} \tag{31}$$

The parameter ρ_Z lies between zero and one, $\varepsilon_{Z,t}$ is drawn from a standard normal distribution, and σ_Z is the standard deviation of the innovation.

3.5 Monetary Policy

Our ultimate objective is to again consider and compare the three different targeting rules discussed in Section 2. For the purposes of estimation, however, we assume that the central bank sets policy according to the same Taylor rule described above, but augment this to include a policy innovation, $\varepsilon_{i,t}$, which is drawn from a standard normal distribution with standard deviation σ_i:

$$\ln(1+i_t) = (1-\rho_i)\ln(1+i^*) + \rho_i \ln(1+i_{t-1}) + \phi_\pi \ln(1+\pi_t) + \phi_x \ln X_t + \phi_y \ln\left(\frac{Y_t}{Y_{t-1}}\right) + \sigma_i \varepsilon_{i,t}. \tag{32}$$

3.6 Aggregation

Since we assume that the government issues no debt, in equilibrium bond-holding is always zero, $B_t = 0$. Combining this with the definition for firm profit and the government's budget constraint gives rise to the aggregate resource constraint:

$$Y_t = C_t + I_t + G_t + \left[\gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2\right] K_t. \tag{33}$$

The expressions for the aggregate price and wage indexes are similar to Section 2, but account for indexation:

$$P_t^{1-\epsilon_p} = (1-\theta_p)P_t^{\#,1-\epsilon_p} + \theta_p(1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)}P_{t-1}^{1-\epsilon_p} \tag{34}$$

$$w_t^{1-\epsilon_w} = (1-\theta_w)w_t^{\#,1-\epsilon_w} + \theta_w(1+\pi_t)^{\epsilon_w-1}(1+\pi_{t-1})^{\zeta_w(1-\epsilon_w)}w_{t-1}^{1-\epsilon_w}. \tag{35}$$

The aggregate production function is given by:

$$Y_t = \frac{A_t\left(u_t K_t\right)^{\alpha} N_t^{1-\alpha} - F}{v_t^p}. \tag{36}$$

The variable v_t^p is the same measure of price dispersion as in the simpler model, defined by (17).

Aggregate welfare is defined similarly as in Section 2, but takes into account internal habit formation:

$$\mathbb{W}_t = \nu_t\left[\ln\left(C_t - bC_{t-1}\right) - \psi v_t^w \frac{N_t^{1+\eta}}{1+\eta}\right] + \beta E_t \mathbb{W}_{t+1}. \tag{37}$$

The variable v_t^w is again a measure of wage dispersion, defined above by (19).

4 Quantitative Analysis

In this section we analyze the welfare properties of nominal GDP targeting, inflation targeting, output gap targeting, and the Taylor rule in the medium scale model. Quantitatively evaluating different policy regimes requires selecting values for the parameters of the model. We calibrate some parameters to match long run moments of the data and estimate the remaining parameters so as to ensure that the model provides an empirically realistic fit to observed data.

The calibrated parameters and their values are listed in Table 5. As in the simpler model of Section 2, the discount factor is set to $\beta = 0.995$, and the elasticities of substitution for goods and labor, ϵ_p and ϵ_w, are set to 10. The steady state level of government spending is chosen so that the steady state ratio of government spending to output is $G^*/Y^* = 0.20$. The scaling parameter on the disutility of labor is set to $\psi = 6$, which implies steady state labor hours of between one-third and one-half for reasonable values of the Frisch elasticity and habit persistence parameter. To first order, this scaling parameter is irrelevant for equilibrium dynamics. The parameter on the linear term in the utilization adjustment cost function, γ_1, is chosen to be consistent with a steady state normalization of utilization to one. Capital's share is set to $\alpha = 1/3$, and the depreciation rate on physical capital is $\delta = 0.025$. The fixed cost, F, is set so that profits are zero in steady state.

The remaining parameters are estimated via Bayesian maximum likelihood. The observable variables in the estimation are the log first differences of real output, real consumption, real investment, the inflation rate, and the nominal interest. To facilitate comparison with the model, we define nominal GDP as the sum of consumption (non-durables and services consumption), investment (the sum of durables consumption and private fixed investment), and government spending (government consumption expenditures and gross investment). These series are all taken from the NIPA tables. We deflate nominal output, as well as the individual components, by the GDP implicit price deflator. We then divide by the civilian non-institutionalized population, take logs, and first difference. Inflation is defined as the log first difference of the GDP deflator. Our measure of the nominal interest rate is the effective Federal Funds Rate, aggregated to a quarterly frequency by averaging monthly observations. We use data from the first quarter of 1984 through the third quarter of 2007. The start date is chosen to account for the large break in volatility associated with the Great Moderation, while the end date is chosen so as to exclude the recent zero lower bound period.

Table 5: Calibrated Parameters, Medium Scale Model

Parameter	Description	Value
β	Discount rate	0.995
ψ	Labor disutility	6
ϵ_w	Elasticity of substitution – Labor	10
ϵ_p	Elasticity of substitution – Goods	10
G^*	SS Government Spending	$G^*/Y^* = 0.2$
α	Capital Share	1/3
F	Fixed cost	$\Pi^* = 0$
γ_1	Utilization linear cost	$u^* = 1$
δ	Depreciation rate	0.025

Notes: The table shows the values of the calibrated parameters in the medium scale model.

Table 6 presents the prior and posterior distributions of the estimated parameters. These results are broadly consistent with Smets and Wouters (2007) and Justiniano et al. (2010). There is significant rigidity in both prices and wages. The average duration between prices changes is roughly three quarters, while the average duration between wage changes is about a year. The latter is consistent with the micro evidence presented in Barattieri et al. (2014). We find moderate levels of wage and price indexation. The investment adjustment cost parameter, τ, and the parameter governing internal habit formation, b, imply significant real inertia. The estimate of the parameter η implies a Frisch labor supply elasticity of about two-thirds. The monetary policy rule features significant inertia and a large response to

inflation (the implied long run response of the interest rate to inflation is about four). The policy rule features a positive response to output growth and no response to the output gap, the latter of which is estimated precisely at zero. This is consistent with the evidence in Coibion and Gorodnichenko (2011) that the Fed moved away from responding to the output gap and toward a much stronger reaction to output growth in the post-Volcker period.

Table 6: Estimated Parameters

	Prior			Posterior			
Parameter	Dist.	Mean	SD	Mode	Mean	SD	90% Probability Interval
b	Beta	0.70	0.10	0.7508	0.7606	0.0713	[0.6634, 0.8759]
τ	Normal	4.00	0.20	3.9676	3.9927	0.2007	[3.6661, 4.3288]
η	Normal	1.50	0.20	1.6244	1.6290	0.2165	[1.3075, 1.9141]
γ_2	Beta	0.10	0.10	0.3783	0.4143	0.1226	[0.2372, 0.6042]
θ_p	Beta	0.70	0.10	0.6762	0.6852	0.0556	[0.6008, 0.7659]
θ_w	Beta	0.70	0.10	0.7887	0.7701	0.0780	[0.6517, 0.9054]
ζ_p	Beta	0.50	0.10	0.3236	0.3417	0.0894	[0.1982, 0.4783]
ζ_w	Beta	0.50	0.10	0.5252	0.5263	0.1071	[0.3610, 0.6990]
ρ_i	Beta	0.70	0.10	0.8814	0.8804	0.0197	[0.8470, 0.9135]
ϕ_π	Normal	0.50	0.10	0.4495	0.4540	0.0478	[0.3747, 0.5330]
ϕ_x	Normal	0.05	0.10	0.0000	0.0000	0.0000	[0.0000, 0.0000]
ϕ_y	Normal	0.20	0.10	0.1469	0.1504	0.0276	[0.1036, 0.1971]
ρ_A	Beta	0.70	0.10	0.8472	0.8295	0.0485	[0.7506, 0.9125]
ρ_ν	Beta	0.70	0.10	0.6916	0.6579	0.1201	[0.4885, 0.8276]
ρ_Z	Beta	0.70	0.10	0.6820	0.6733	0.0575	[0.5840, 0.7708]
ρ_G	Beta	0.70	0.10	0.9506	0.9488	0.0001	[0.9267, 0.9706]
σ_A	Inv. Gamma	0.01	0.01	0.0064	0.0073	0.0015	[0.0046, 0.0100]
σ_ν	Inv. Gamma	0.01	0.01	0.0181	0.0203	0.0041	[0.0128, 0.0282]
σ_Z	Inv. Gamma	0.01	0.01	0.0389	0.0405	0.0040	[0.0330, 0.0468]
σ_G	Inv. Gamma	0.01	0.01	0.0098	0.0100	0.0001	[0.0087, 0.0113]
σ_i	Inv. Gamma	0.002	0.01	0.0013	0.0013	0.0001	[0.0011, 0.0015]

Notes: The variables used in the estimation are the growth rates of output, consumption, and investment, and the levels of inflation and the nominal interest rate. Construction of these series is as described in the text. The posterior is generated with 100,000 Metropolis-Hastings draws.

The estimated model produces second moments which align closely with their counterparts in the data. Consumption growth is less volatile than output growth, while investment growth is about three times more volatile than output growth. Output, consumption, and investment are strongly positively correlated, while the nominal interest rate is roughly uncorrelated with

output growth and the inflation rate is mildly negatively correlated with output growth. The first order autocorrelations of output, consumption, and investment growth are all significantly positive. In terms of contributions to business cycle dynamics, the investment shock is the most important disturbance. It accounts for about 50 percent of the unconditional variance of output growth. The productivity shock explains about 30 percent of the variance of output growth. The preference and government spending shocks each contribute a little less than 10 percent, while the monetary shock accounts for about 5 percent of the unconditional variance of output growth. This variance decomposition is very much in line with the results in Justiniano et al. (2010).

4.1 Evaluating Monetary Policy Rules

We solve the medium scale model using the calibrated or estimated values listed in Tables 5 and 6 using a second order approximation about the non-stochastic steady state. Compensating variations are constructed comparing average welfare under different policy regimes to the hypothetical flexible price and wage equilibrium. These exercises are identical to the ones carried out in Section 2. When evaluating the welfare performance of the estimated Taylor rule, we set the standard deviation of the policy innovation to zero so as to facilitate comparison with the other targeting rules, which feature no shocks.

The compensating variations for the different monetary policy rules in the estimated model are summarized in the first main column of Table 7. The relative performance of the different targeting rules is the same as in the small scale model from Section 2. The welfare loss associated with nominal GDP targeting is 0.11 percent of consumption. This is not as good as output gap targeting, which has a welfare loss of 0.03 percent of consumption, but outperforms the estimated Taylor rule (welfare loss of 0.24 percent of consumption) and does significantly better than inflation targeting, which produces a compensating variation of 0.86 percent of consumption.

The remaining columns of Table 7 consider different values of the parameters governing price and wage stickiness. For these exercises all but the listed parameter are fixed at their estimated or calibrated values. The patterns in the Table again echo those from the small scale model. If either prices or wages are completely flexible, the output gap targeting rule implements the flexible price and wage allocation with a compensating variation of zero. If wages are perfectly flexible then inflation targeting also implements the flexible price allocation. Nominal GDP targeting performs very well, producing a lower welfare loss than the Taylor rule in all specifications and outperforming inflation targeting by a wide margin in all but the case where wages are flexible. As in the small scale model, the relative performance

of nominal GDP targeting to inflation targeting depends on the relative stickiness of wages to prices.

Table 7: Consumption Equivalent Welfare Losses from Different Policy Rules
Medium Scale Model

	Estimated Model	$\theta_p = 0.00$	$\theta_p = 0.50$	$\theta_w = 0.50$	$\theta_w = 0.00$
NGDP	0.1056	0.0950	0.0976	0.1276	0.0755
Inflation	0.8630	0.8630	0.8630	0.6182	0.0000
Output Gap	0.0343	0.0000	0.0277	0.0312	0.0000
Taylor Rule	0.2439	0.1998	0.2325	0.1286	0.1132

Notes: The table contains the compensating variations of three different targeting rules as well as a Taylor rule in the medium scale model. The column labeled "Estimated Model" uses the parameter values described in Tables 5 and 6. When evaluating the compensating variation for the Taylor rule the standard deviation of the monetary policy shock is set to zero. The other columns describe values of the price or wage rigidity parameters. For the exercises described in these columns all other parameters are fixed at their calibrated or estimated values.

In Table 8 we compare the compensating variations for different policy rules conditional on specific shocks. For these exercises we solve for the standard deviation of the listed shock that would generate the same volatility of log output as in the baseline model when all other shocks have standard deviations of zero. The output gap targeting rule produces the smallest welfare losses conditional on each of the four considered shocks. Nominal GDP targeting is the second best rule conditional on three of the four shocks, with the exception being the government spending shock. Though nominal GDP targeting produces a fairly high compensating variation conditional on the government spending shock, this shock is estimated to be a relatively unimportant driver of fluctuations. Inflation targeting does very poorly conditional on productivity shocks, but produces fairly small compensating variations conditional on investment and preference shocks. The Taylor rule performs fairly well conditional on all but the preference shock.

In the estimated model the productivity shock is a "supply" shock in that it moves output and inflation in opposite directions, while the other three shocks are "demand" shocks in that they result in output and inflation co-moving positively (these statements are conditional on the estimated model where policy is governed by the estimated Taylor rule). In the medium scale model there is not a clean demarcation between "demand" and "supply" because the investment, preference, and government spending shocks would affect output when prices and wages are flexible. Nevertheless, some of the intuition from the simple model does carry over to this setting. In particular, the difference between nominal GDP and inflation

targeting is starkest conditional on the productivity shock, with inflation targeting producing a compensating variation more than fifteen times bigger than nominal GDP targeting. Further, the welfare differences between nominal GDP and inflation targeting are smaller conditional on the three demand shocks. Unlike the simple model, inflation targeting does not implement the flexible price and wage allocation conditional on these three shocks, and nominal GDP targeting outperforms inflation targeting conditional on two of the three demand shocks.

Table 8: Consumption Equivalent Welfare Losses from Different Policy Rules, Shock Specific
Medium Scale Model

	Shocks			
	Productivity	Investment	Preference	Government Spending
NGDP	0.1141	0.0872	0.2012	0.5507
Inflation	1.7684	0.1266	0.3379	0.4828
Output Gap	0.0684	0.0062	0.0173	0.0209
Taylor Rule	0.1562	0.2779	1.1869	0.1075

Notes: The table contains the compensating variations of three different targeting rules as well as a Taylor rule in the medium scale model. We solve for the standard deviation of the listed shock that would generate the same volatility of log output as in the baseline model when all other shocks have standard deviations of zero. The resulting shock standard deviations are $\sigma_A = 0.0096$ for the productivity shock only case, $\sigma_Z = 0.0537$ when there are only investment shocks, $\sigma_\nu = 0.1175$ when conditioning on only preference shocks, and $\sigma_G = 0.1103$ for government spending shocks. All other parameters are fixed at their calibrated or estimated values.

We consider some robustness to different values of parameters unrelated to the shock processes and nominal price and wage stickiness in Table 9.[8] For these exercises all parameters but the one listed in the column are set to their baseline values. The basic pattern of results is consistent with our previous analysis – the output gap targeting rule is always the best-performing rule, while nominal GDP targeting is typically the second best. When there are no investment adjustment costs ($\tau = 0$), an infinite Frisch labor supply elasticity ($\eta = 0$), or no variable utilization ($\gamma_2 = \infty$), nominal GDP targeting performs slightly worse and inflation targeting a little better than in the baseline estimated model, though nominal GDP targeting is still more desirable than inflation targeting by a wide margin. The compensating variations for both nominal GDP targeting and inflation targeting are slightly smaller with no habit

[8]We consider several additional robustness exercises which are relegated to Appendix C. In Table 10 we compute the compensating variations for the different policy rules when all estimated parameters are set at either the 10th or 90th percentiles of the posterior distribution. In Tables 11 and 12 we set each parameter to either the 10th or 90th percentile of the posterior distribution, respectively, while holding all other parameters fixed at the posterior mode, and compute compensating variations for the different policy rules. Qualitatively the relative ranking of different policy rules obtains in all the different parameter configurations considered.

formation ($b = 0$) and lower elasticities of substitution ($\epsilon_p = \epsilon_w = 5$), though again nominal GDP targeting is preferred to inflation targeting. The Taylor rule produces compensating variations similar to the baseline estimated model in all cases except where there are no investment adjustment costs, in which case it performs very poorly. Assuming full indexation of non-updated prices and wages to lagged inflation ($\gamma_p = \gamma_w = 1$) makes very little difference for any of the welfare losses associated with different policies relative to the baseline estimated model.

Table 9: Consumption Equivalent Welfare Losses from Different Policy Rules, Parameter Robustness
Medium Scale Model

	$\tau = 0$	$b = 0$	$\eta = 0$	$\gamma_2 = \infty$	$\epsilon_p = \epsilon_w = 5$	$\gamma_p = \gamma_w = 1$
NGDP	0.2204	0.0919	0.2051	0.1056	0.0745	0.3020
Inflation	0.6303	0.4683	0.4577	0.8021	0.6240	0.8630
Output Gap	0.0403	0.0292	0.0374	0.0346	0.0141	0.0265
Taylor Rule	5.0947	0.2654	0.0996	0.2463	0.1093	0.3152

Notes: The table contains the compensating variations of three different targeting rules as well as a Taylor rule in the medium scale model for different values of select parameters. The value of a parameter(s) different than in our baseline is indicated in each column; the remaining parameters are set at their calibrated or estimated values listed in Tables 5 and 6.

4.2 Gap or NGDP Targeting: Measurement Difficulties

In both the small and medium scale versions of the model, we find that output gap targeting is the most desirable policy rule. Nominal GDP targeting typically performs very well, in many instances almost as well as gap targeting. A potential advantage of nominal GDP targeting over gap targeting is that the former is easily observed, whereas the gap is based on a hypothetical model construct that is likely difficult to observe even ex-post, much less in real time. A nominal GDP target is likely also far easier to communicate to the public than a gap targeting rule.

There exists a large literature that studies the difficulties of observing the output gap and the implications for monetary policy. Orphanides and van Norden (2002) find that the measurement of the output gap in real time is acutely affected by the method of detrending. Moreover, the ex post revisions in the output gap are on the same order of magnitude as the output gap itself. Combing this uncertainty about the trend with conventional measurement error implies that the real time measurement of the output gap is an unreliable measure of the true output gap. Orphanides (2001) shows that inputting real time data in to a Taylor rule with standard coefficients consistently under predicts the Fed Funds rate and Orphanides

(2003) shows that smaller coefficients in the Taylor rule are in fact welfare improving in the presence of measurement error. Since any reaction to observed deviations of the output gap and inflation is in fact a reaction to their true deviations plus measurement error, the optimally behaved central bank is more prudent. In recent work, Beckworth and Hendrickson (2015) show that a rule where the nominal interest rate is a linear function of nominal GDP growth outperforms the conventional Taylor rule when real time forecasts are used for the output gap. This corroborates the results of Orphanides (2003), who finds that policy rules that respond to growth rates of economic variables rather than their misperceived deviations from their natural rates are welfare improving.

We consider a couple of different exercises to examine the relative desirability of gap versus nominal GDP targeting when the gap is difficult to observe. In the first, we assume that the central bank observes the flexible price and wage level of output with noise. In particular, we assume:

$$\ln Y_t^{f,obs} = \ln Y_t^f + n_t. \tag{38}$$

In this specification Y_t^f is the flexible price and wage level of output, while $Y_t^{f,obs}$ is the central bank's observed flexible price and wage level of output. The two differ by n_t, which is a noise process which we assume obeys a mean-zero stationary AR(1) process:

$$n_t = \rho_n n_{t-1} + \sigma_n \varepsilon_{n,t} \tag{39}$$

Here $0 < \rho_n < 1$ is a persistence parameter, $\varepsilon_{n,t}$ is a shock drawn from a standard normal distribution, and σ_n is the standard deviation of the shock. These noise shocks capture in a tractable yet realistic way the idea that the central bank may have difficulty in observing the true flexible price and wage level of output. The central bank's observed output gap is the ratio of the actual level of output to the observed flexible price and wage level of output, e.g. $X_t = Y_t / Y_t^{f,obs}$. Noise shocks cause an output gap targeting central bank to target the wrong measure of the gap. One would expect that gap targeting would be less desirable the noisier is the observed flexible price and wage level of output.

We consider the following experiment. We find the pairs of (ρ_n, σ_n) for which gap targeting (with the mis-measured output gap) and nominal GDP targeting produce the same welfare loss. Figure 2 plots in the left panel these combinations for which a central bank would be indifferent between gap and nominal GDP targeting. At parameter combinations above this frontier (areas shaded green) nominal GDP targeting is preferred to gap targeting. The right panel plots the overall standard deviation of noise (e.g. the standard deviation of n_t) at the (ρ_n, σ_n) pairs plotted in the left panel. Areas shaded green represent standard deviations of the noise process for which nominal GDP targeting produces a lower welfare loss than does

gap targeting.

The standard deviation of the noise shock to the observed flexible price and wage level of output which would equate the welfare losses of nominal GDP and gap targeting is always less than 0.66 percent for any value of the persistence parameter. For levels of persistence in excess of 0.9 the amount of noise required to equate the two targeting rules is less than 0.25 percent. It is interesting to note that the relationship between ρ_n and the standard deviation of the noise shock which equates gap targeting to NGDP targeting is non-monotonic – for low values of persistence, as ρ_n increases the standard deviation of the noise shock increases. We surmise that this has to do with a smoothing motive – if the noise shocks are i.i.d., then the central bank's perceived flexible price and wage level of output fluctuates substantially period-to-period, imparting significant volatility to interest rates. For very high levels of persistence, however, the amount of noise required to equate nominal GDP and gap targeting is smaller the bigger is ρ_n. The right panel plots the overall standard deviation of the noise process for the parameter combinations in the left panel. The overall standard deviation of the noise process necessary for output gap and nominal GDP targeting to produce the same compensating variations is always less than 1 percent, and is often substantially less than that. To put this number into perspective, the standard deviation of the log flexible price and wage level of output in the estimated model is 1.8 percent. Roughly speaking, if volatility of noise is greater than 1/3-1/2 the volatility of the flexible price and wage level of output itself, nominal GDP targeting dominates output gap targeting.

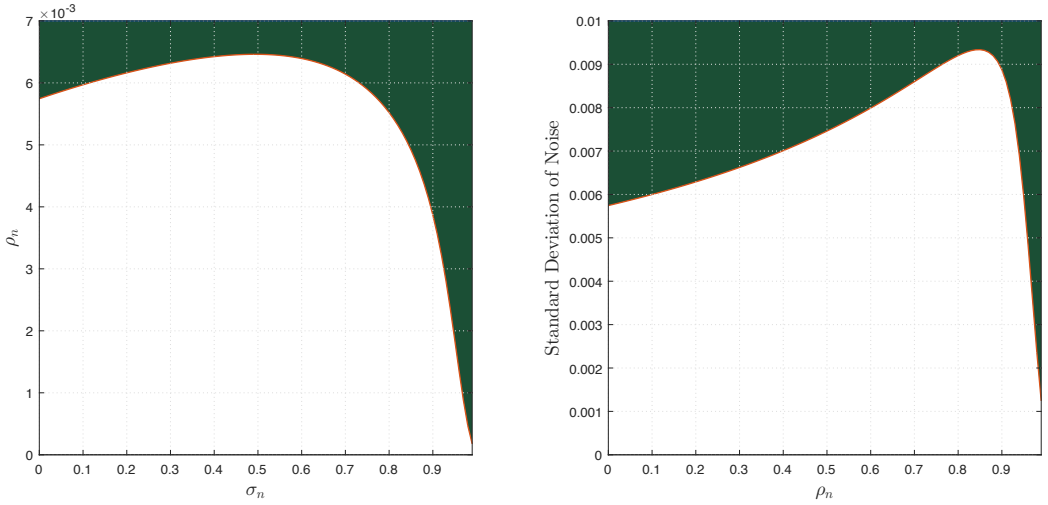

Figure 2: Measurement Error and Gap Targeting vs. NGDP Targeting

Notes: The left panel of the figure plots the combinations of (ρ_n, σ_n) for which output gap and nominal GDP targeting produce the same compensating variation in the estimated medium scale model. The right panel plots the standard deviation of the noise process, n_t, at the parameter values plotted in the left panel. The areas shaded green correspond to parameter configurations where NGDP targeting is preferred to gap targeting.

We also consider a second specification in which, rather than being observed with noise, the central bank's perceived measure of the flexible price and wage level of output adjusts slowly to actual movements in the flexible price and wage level of output. In particular:

$$\ln Y_t^{f,obs} = \omega \ln Y_{t-1}^{f,obs} + (1-\omega) \ln Y_t^f. \tag{40}$$

The parameter ω is restricted to lie strictly between 0 and 1. In this specification ω governs the speed with which the flexible price and wage level of output adjusts to its true level. A specification like this could be motivated as reduced form way to model an optimal filtering problem solved by a central bank. As $\omega \to 1$, this rule would be equivalent to an output targeting rule, since the perceived flexible price and wage level of output would always equal its non-stochastic steady state, which is the same as the steady state level of actual output.

In Figure 3 we calculate the compensating variation of gap targeting (when the gap is constructed using the observed flexible price and wage level of output) for different values of the parameter ω. For comparison, we also show the compensating variation associated with nominal GDP targeting.

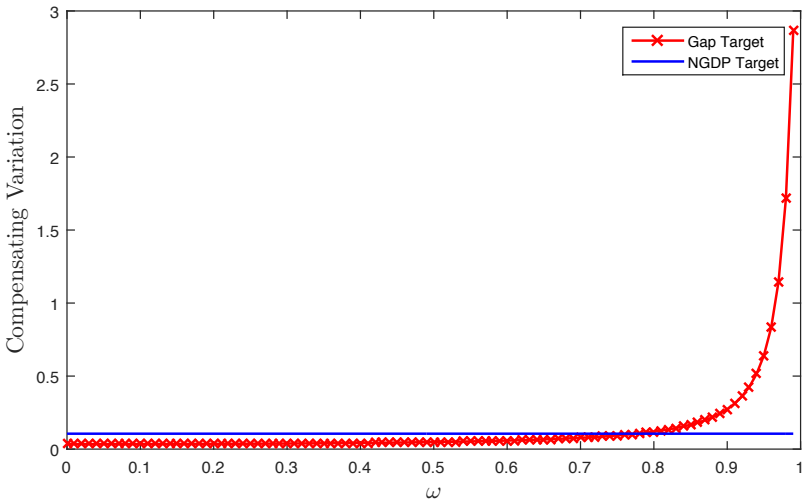

Figure 3: Slow-Moving Gap Target

Notes: The solid blue line plots the compensating variation associated with nominal GDP targeting in the estimated medium scale model. The ride line marked with "x" plots the compensating variation associated with gap targeting for different values of the parameter ω when the perceived flexible price and wage level of output follows the process described in (40).

Output gap targeting produces a smaller welfare loss than nominal GDP targeting for values of ω less than about 0.8. This is fairly high and suggests that the perceived flexible price and wage level of output must adjust slowly to its true value for nominal GDP targeting to be preferred to gap targeting. Nevertheless, there is significant downside risk to output

gap targeting in the event that ω is high. The compensating variation associated with gap targeting rises quickly at values of ω above 0.8. Once above about $\omega = 0.9$ or so output gap targeting performs worse than inflation targeting, and produces compensating variations in excess of 1 percent. Since when $\omega \to 1$ this specification is equivalent to an output target, these results are consistent with the analysis in Schmitt-Grohe and Uribe (2007), who find that it is non-optimal to target the deviation of output from its steady state in a Taylor rule.

4.3 Gap vs NGDP Targeting: Equilibrium Determinacy

There exists a literature which emphasizes that a central bank strongly responding to the output gap could result in equilibrium indeterminacy. This point has been made in the context of the Taylor rule in a sticky price New Keynesian model by Hornstein and Wolman (2005), Ascari and Ropele (2009), and Coibion and Gorodnichenko (2011). These authors show that the standard Taylor principle for a Taylor rule to support a determinate equilibrium may break down when trend inflation is positive. In particular, strong responses to the output gap can result in equilibrium indeterminacy even if the long run response of the interest rate to inflation is greater than one. Furthermore, the minimum required response to inflation is increasing in the coefficient on the gap for positive levels of trend inflation.

Whereas these papers focus on determinacy within the context of an instrument rule, to our knowledge no paper has studied the issue of determinacy with trend inflation for strict targeting rules. We find that gap targeting results in an indeterminate equilibrium at trend inflation rates higher than 0.2 percent annualized, which is far below the observed average level of inflation of about 2 percent over the last thirty years. Nominal GDP targeting, in contrast, supports a determinate equilibrium for any level of trend inflation. This is because in the long run nominal GDP targeting is isomorphic to a price level target, which always ensures equilibrium determinacy.[9]

5 Conclusion

The design of monetary policy has been the subject of a voluminous and influential literature. In spite of widespread discussion in the press and policy circles, the normative properties of nominal GDP targeting have not been subject to scrutiny within the context of the quantitative frameworks commonly used at central banks and among academic macroe-

[9]We also find that inflation targeting supports a determinate equilibrium for any level of trend inflation. Though we do not report the results here, we continue to find that nominal GDP targeting produces significantly lower welfare losses than inflation targeting (and the Taylor rule) in the model when trend inflation is positive.

conomists. The objective of this paper has been to analyze the welfare properties of nominal GDP targeting in comparison to other popular policy rules in an empirically realistic New Keynesian model with both price and wage rigidity.

We find that nominal GDP targeting performs well in this model. It typically produces small welfare losses and comes close to fully implementing the flexible price and wage allocation. It produces smaller welfare losses than an estimated Taylor rule and significantly outperforms inflation targeting. It tends to perform best relative to these alternative rules when wages are sticky relative to prices and conditional on supply shocks. While output gap targeting always at least weakly outperforms nominal GDP targeting, the differences in welfare losses associated with the two rules are small. Nominal GDP targeting may produce lower welfare losses than gap targeting if the central bank has difficulty measuring the output gap in real time. Nominal GDP targeting always supports a determinate equilibrium, whereas output gap targeting may result in indeterminacy if trend inflation is positive. Overall, our analysis suggests that nominal GDP targeting is a policy alternative that central banks ought to take seriously.

There are a number of possible extensions of our analysis. Two which immediately come to mind are financial frictions and the zero lower bound. Though our medium scale model includes investment shocks, which have been interpreted as a reduced form for financial shocks in Justiniano et al. (2011), it would be interesting to formally model financial frictions and examine how nominal GDP targeting interacts with those. Second, our analysis abstracts from the zero lower bound on nominal interest rates. It would be interesting to study how a commitment to a nominal GDP target might affect the frequency, duration, and severity of zero lower bound episodes.

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A Basic New Keynesian Model

The equations below characterize the equilibrium of the simple New Keynesian model described in Section 2.

$$\frac{1}{C_t} = \beta E_t \frac{\nu_{t+1}}{\nu_t} \frac{1}{C_{t+1}} \frac{1+i_t}{1+\pi_{t+1}} \tag{41}$$

$$w_t^{\#} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{H_{1,t}}{H_{2,t}} \tag{42}$$

$$H_{1,t} = \psi \nu_t \left(\frac{w_t^{\#}}{w_t}\right)^{-\epsilon_w(1+\eta)} N_t^{1+\eta} + \theta_w \beta E_t \left(\frac{w_{t+1}^{\#}}{w_t^{\#}}\right)^{\epsilon_w(1+\eta)} (1+\pi_{t+1})^{\epsilon_w(1+\eta)} H_{1,t+1} \tag{43}$$

$$H_{2,t} = \nu_t \frac{1}{C_t} \left(\frac{w_t^{\#}}{w_t}\right)^{-\epsilon_w} N_t + \theta_w \beta E_t \left(\frac{w_{t+1}^{\#}}{w_t^{\#}}\right)^{\epsilon_w} (1+\pi_{t+1})^{\epsilon_w - 1} H_{2,t+1} \tag{44}$$

$$mc_t = \frac{w_t}{A_t} \tag{45}$$

$$\frac{1+\pi_t^{\#}}{1+\pi_t} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1,t}}{X_{2,t}} \tag{46}$$

$$X_{1,t} = \nu_t \frac{1}{C_t} mc_t Y_t + \theta_p \beta E_t (1+\pi_{t+1})^{\epsilon_p} X_{1,t+1} \tag{47}$$

$$X_{2,t} = \nu_t \frac{1}{C_t} Y_t + \theta_p \beta E_t (1+\pi_{t+1})^{\epsilon_p - 1} X_{2,t+1} \tag{48}$$

$$Y_t = C_t \tag{49}$$

$$Y_t = \frac{A_t N_t}{v_t^p} \tag{50}$$

$$(1+\pi_t)^{1-\epsilon_p} = (1-\theta_p)(1+\pi_t^{\#})^{1-\epsilon_p} + \theta_p \tag{51}$$

$$w_t^{1-\epsilon_w} = (1-\theta_w) w_t^{\#,1-\epsilon_w} + \theta_w (1+\pi_t)^{\epsilon_w - 1} w_{t-1}^{1-\epsilon_w} \tag{52}$$

$$v_t^p = (1+\pi_t)^{\epsilon_p} \left[(1-\theta_p)(1+\pi_t^{\#})^{-\epsilon_p} + \theta_p v_{t-1}^p\right] \tag{53}$$

$$\ln(1+i_t) = (1-\rho_i)\ln(1+i^*) + \rho_i \ln(1+i_{t-1}) + \phi_\pi \ln(1+\pi_t) + \phi_y \ln\left(\tfrac{Y_t}{Y_{t-1}}\right)$$

or $$\pi_t = 0 \tag{54}$$

or $$P_t Y_t = (PY)^*$$

$$\ln A_t = \rho_A \ln A_{t-1} + \sigma_A \varepsilon_{A,t} \tag{55}$$

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \sigma_\nu \varepsilon_{\nu,t} \tag{56}$$

$$\mathbb{W}_t = \nu_t \left[\ln C_t - \psi v_t^w \frac{N_t^{1+\eta}}{1+\eta} \right] + \beta E_t \mathbb{W}_{t+1} \tag{57}$$

$$v_t^w = (1-\theta_w) \left(\frac{w_t^\#}{w_t} \right)^{-\epsilon_w(1+\eta)} + \theta_w \left(\frac{w_t}{w_{t-1}} (1+\pi_t) \right)^{\epsilon_w(1+\eta)} v_{t-1}^w \tag{58}$$

Equation (41) is the consumption Euler equation and equations (42) - (44) describe optimal wage-setting for households given the opportunity to adjust their wage. Equation (45) describes real marginal cost and comes from firms minimizing cost. Equations (46) - (48) describe optimal price-setting. Here we have defined $1 + \pi_t^\# = \frac{P_t^\#}{P_{t-1}}$ as reset price inflation. Equation (49) is the aggregate resource constraint and (50) is the aggregate production function. Equations (51) and (52) describe the evolution of aggregate inflation and the aggregate real wage, respectively. The process for price dispersion is given by (53). Monetary policy is characterized by (54), which either follows the Taylor rule, inflation targeting, or nominal GDP targeting. Equations (55) and (56) are the stochastic processes for the productivity and preference shocks. Aggregate welfare obeys (57), while the expression for wage dispersion is given by (58). Given initial values of the state variables and current realizations of the innovations in the shock processes, an equilibrium consists of a non-explosive sequence of $\{C_t, i_t, \pi_t, \nu_t, w_t^\#, H_{1,t}, H_{2,t}, w_t, N_t, mc_t, A_t, \pi_t^\#, X_{1,t}, X_{2,t}, v_t^p, \mathbb{W}_t, v_t^w, Y_t\}$ such that equations (41)-(58) all hold.

B Medium Scale Model

The equations below characterize the equilibrium of the medium scale model described in Section 3.

$$\lambda_t = \frac{\nu_t}{C_t - bC_{t-1}} - \beta b E_t \frac{\nu_{t+1}}{C_{t+1} - bC_t} \tag{59}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}} \tag{60}$$

$$R_t = \gamma_1 + \gamma_2 (u_t - 1) \tag{61}$$

$$\mu_t = \beta E_t \lambda_{t+1} \left[R_{t+1} u_{t+1} - \left(\gamma_1 (u_{t+1} - 1) + \frac{\gamma_2}{2} (u_{t+1} - 1)^2 \right) \right] + \beta (1-\delta) E_t \mu_{t+1} \tag{62}$$

$$\lambda_t = \mu_t Z_t \left[1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \tau \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \beta \tau E_t \mu_{t+1} Z_{t+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \tag{63}$$

$$w_t^\# = \frac{\epsilon_w}{\epsilon_w - 1} \frac{H_{1,t}}{H_{2,t}} \tag{64}$$

$$H_{1,t} = \psi\nu_t \left(\frac{w_t^{\#}}{w_t}\right)^{-\epsilon_w(1+\eta)} N_t^{1+\eta} + \theta_w\beta E_t \left(\frac{w_{t+1}^{\#}}{w_t^{\#}}\right)^{\epsilon_w(1+\eta)} (1+\pi_t)^{-\zeta_w\epsilon_w(1+\eta)}(1+\pi_{t+1})^{\epsilon_w(1+\eta)} H_{1,t+1} \quad (65)$$

$$H_{2,t} = \lambda_t \left(\frac{w_t^{\#}}{w_t}\right)^{-\epsilon_w} N_t + \theta_w\beta E_t \left(\frac{w_{t+1}^{\#}}{w_t^{\#}}\right)^{\epsilon_w} (1+\pi_t)^{\zeta_w(1-\epsilon_w)}(1+\pi_{t+1})^{\epsilon_w-1} H_{2,t+1} \quad (66)$$

$$\frac{u_t K_t}{N_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{R_t} \quad (67)$$

$$mc_t = \frac{w_t^{1-\alpha} R_t^{\alpha}}{A_t}(1-\alpha)^{\alpha-1}\alpha^{-\alpha} \quad (68)$$

$$\frac{1+\pi_t^{\#}}{1+\pi_t} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{X_{1,t}}{X_{2,t}} \quad (69)$$

$$X_{1,t} = \lambda_t mc_t Y_t + \theta_p\beta E_t(1+\pi_t)^{-\zeta_p\epsilon_p}(1+\pi_{t+1})^{\epsilon_p} X_{1,t+1} \quad (70)$$

$$X_{2,t} = \lambda_t Y_t + \theta_p\beta E_t(1+\pi_t)^{\zeta_p(1-\epsilon_p)}(1+\pi_{t+1})^{\epsilon_p-1} X_{2,t+1} \quad (71)$$

$$Y_t = C_t + I_t + G_t + \left[\gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2\right] K_t \quad (72)$$

$$K_{t+1} = Z_t\left[1 - \frac{\tau}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1-\delta)K_t \quad (73)$$

$$Y_t = \frac{A_t \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha} - F}{v_t^p} \quad (74)$$

$$(1+\pi_t)^{1-\epsilon_p} = (1-\theta_p)(1+\pi_t^{\#})^{1-\epsilon_p} + \theta_p(1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)} \quad (75)$$

$$w_t^{1-\epsilon_w} = (1-\theta_w) w_t^{\#,1-\epsilon_w} + \theta_w(1+\pi_t)^{\epsilon_w-1}(1+\pi_{t-1})^{\zeta_w(1-\epsilon_w)} w_{t-1}^{1-\epsilon_w} \quad (76)$$

$$v_t^p = (1+\pi_t)^{\epsilon_p}\left[(1-\theta_p)(1+\pi_t^{\#})^{-\epsilon_p} + \theta_p(1+\pi_{t-1})^{-\epsilon_p\zeta_p} v_{t-1}^p\right] \quad (77)$$

$$\ln(1+i_t) = (1-\rho_i)\ln(1+i^*) + \rho_i \ln(1+i_{t-1}) + \phi_\pi \ln(1+\pi_t) + \phi_y \ln\left(\tfrac{Y_t}{Y_{t-1}}\right)$$
$$\text{or} \quad \pi_t = 0 \quad (78)$$
$$\text{or} \quad P_t Y_t = (PY)^*$$

$$\ln A_t = \rho_A \ln A_{t-1} + \sigma_A \varepsilon_{A,t} \quad (79)$$

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \sigma_\nu \varepsilon_{\nu,t} \quad (80)$$

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \sigma_Z \varepsilon_{Z,t} \quad (81)$$

$$\ln G_t = (1-\rho_G)\ln G^* + \rho_G \ln G_{t-1} + \sigma_G \varepsilon_{G,t} \quad (82)$$

$$\mathbb{W}_t = \nu_t \left[\ln\left(C_t - bC_{t-1}\right) - \psi v_t^w \frac{N_t^{1+\eta}}{1+\eta} \right] + \beta E_t \mathbb{W}_{t+1} \tag{83}$$

$$v_t^w = (1-\theta_w)\left(\frac{w_t^{\#}}{w_t}\right)^{-\epsilon_w(1+\eta)} + \theta_w \left(\frac{w_t}{w_{t-1}}(1+\pi_t)\right)^{\epsilon_w(1+\eta)} (1+\pi_{t-1})^{-\zeta_w \epsilon_w (1+\eta)} v_{t-1}^w \tag{84}$$

In these expressions λ_t is the Lagrange multiplier on the flow budget constraint of a household and μ_t is the multiplier on the capital accumulation equation. Equation (59) defines the marginal utility of income, while (60) is a standard Euler equation for bonds. The first order condition for capital utilization is given by (61). Equation (62) is the Euler equation for physical capital and (63) is the optimality condition for the choice of investment. Equations (64)-(66) describe optimal wage-setting. Expressions (67) and (68) come out of cost-minimization for firms and establish that all firms face the same real marginal cost and hire capital services and labor in the same ratio, equal to the aggregate ratio. Equations (69)-(71) describe optimal price-setting. The aggregate resource constraint is given by (72), the law of motion for physical capital by (73), and the aggregate production function by (74). Equations (75) and (76) describe the evolution of inflation and the real wage. The law of motion for price dispersion is (77). Monetary policy is characterized by one of the three rules in (78). The exogenous processes for productivity, the preference shock, the investment shock, and government spending are given by (79)-(82). Aggregate welfare is given by (83) and the process for wage dispersion is (84). Given initial values of the state variables and current realizations of the innovations in the shock processes, an equilibrium consists of a non-explosive sequence of $\left\{\lambda_t, \nu_t, C_t, i_t, \pi_t, R_t, u_t, \mu_t, Z_t, I_t, w_t^{\#}, H_{1,t}, H_{2,t}, w_t, N_t, K_t, mc_t, \pi_t^{\#}, X_{1,t}, X_{2,t}, G_t, A_t, v_t^p, \mathbb{W}_t, v_t^w, Y_t\right\}$ such that equations (59)–(84) all hold.

C Parameter Robustness

Table 10 shows compensating variations associated with different policy rules in the medium scale model when all parameter values are set at the 10th percentile of the posterior distribution. The 10th percentiles of the posterior distribution of the parameters are shown in Table 6.

Table 10: Consumption Equivalent Welfare Losses from Different Policy Rules
Medium Scale Model, Parameter Robustness

	Estimated Model	10th Percentile, All Parameters	90th Percentile, All Parameters
NGDP	0.1056	0.0424	0.7731
Inflation	0.8630	0.2865	7.3909
Output Gap	0.0343	0.0115	0.1777
Taylor Rule	0.2439	0.0838	2.0121

Notes: The table contains the compensating variations of three different targeting rules as well as a Taylor rule in the medium scale model when all parameters are set to the 10th percentile of the posterior distribution. When evaluating the compensating variation for the Taylor rule the standard deviation of the monetary policy shock is set to zero. The other columns describe values of the price or wage rigidity parameters. For the exercises described in these columns all other parameters are fixed at their calibrated or estimated values.

Table 11 shows compensating variations associated with different policy rules in the medium scale model when parameters are set at the 10th percentile of the posterior distribution one at a time. For this exercise only the listed parameter is set at the 10th percentile of its posterior distribution; all non-listed parameters are set at the mode of the posterior. Because the parameters of the estimated Taylor rule are irrelevant for the compensating variations associated with the different targeting rules, the compensating variations are listed as "n/a" in this portions of the table.

Table 11: Consumption Equivalent Welfare Losses from Different Policy Rules, 10th Percentiles of Parameters Medium Scale Model

	NGDP	Inflation	Output Gap	Taylor rule
b = 0.6634	0.0981	0.7458	0.0329	0.2452
τ = 3.6661	0.1084	0.8432	0.0343	0.255
χ = 1.3075	0.1077	0.7940	0.0348	0.2101
γ_2 = 0.2372	0.1056	0.9000	0.0342	0.2426
θ_p = 0.6008	0.1012	0.863	0.0319	0.2386
θ_w = 0.6517	0.1126	0.7395	0.0332	0.1587
ζ_p = 0.1982	0.1107	0.8630	0.0398	0.2507
ζ_w = 0.3610	0.0920	0.8630	0.0275	0.2618
ρ_i = 0.8470	n/a	n/a	n/a	0.2980
ϕ_π = 0.3747	n/a	n/a	n/a	0.2653
ϕ_x = 0.0000	n/a	n/a	n/a	0.2439
ϕ_y = 0.1036	n/a	n/a	n/a	0.2591
ρ_A = 0.7506	0.0886	0.7350	0.0249	0.2137
ρ_ν = 0.4885	0.1018	0.8575	0.0340	0.2327
ρ_Z = 0.5840	0.0897	0.8388	0.0329	0.1793
ρ_G = 0.9267	0.1046	0.8624	0.0343	0.2438
σ_A = 0.0046	0.0814	0.4887	0.0198	0.2107
σ_ν = 0.0128	0.1032	0.8589	0.0341	0.2299
σ_Z = 0.0330	0.0928	0.8443	0.0334	0.2031
σ_G = 0.0087	0.1047	0.8622	0.0343	0.2437

Notes: The table contains the compensating variations of three different targeting rules as well as a Taylor rule in the medium scale model when each parameter is set at the 10th percentile of its post distribution holding all other parameters fixed at the mode of the posterior.

Table 12 shows compensating variations associated with different policy rules in the medium scale model when parameters are set at the 10th percentile of the posterior distribution one at a time. For this exercise only the listed parameter is set at the 90th percentile of its posterior distribution; all non-listed parameters are set at the mode of the posterior. Because the parameters of the estimated Taylor rule are irrelevant for the compensating variations associated with the different targeting rules, the compensating variations are listed as "n/a" in this portions of the table.

Table 12: Consumption Equivalent Welfare Losses from Different Policy Rules, 90th Percentiles of Parameters Medium Scale Model

	NGDP	Inflation	Output Gap	Taylor rule
b = 0.8759	0.1250	1.0646	0.0373	0.2484
τ = 4.3288	0.1025	0.8861	0.0343	0.2322
η = 1.9141	0.1048	0.9274	0.0339	0.2754
γ_2 = 0.6042	0.1056	0.8400	0.0344	0.2447
θ_p = 0.7659	0.1136	0.8630	0.036	0.2544
θ_w = 0.9054	0.1140	1.3226	0.0347	0.5346
ζ_p = 0.4783	0.1008	0.8630	0.0281	0.2368
ζ_w = 0.6990	0.1346	0.8630	0.0476	0.2384
ρ_i = 0.9100	n/a	n/a	n/a	0.2080
ϕ_π = 0.5330	n/a	n/a	n/a	0.2292
ϕ_x = 0.0000	n/a	n/a	n/a	0.2439
ϕ_y = 0.1971	n/a	n/a	n/a	0.2291
ρ_A = 0.9125	0.1317	1.0937	0.0458	0.3196
ρ_ν = 0.8276	0.1153	0.8755	0.0351	0.2576
ρ_Z = 0.7708	0.1346	0.9103	0.0371	0.3628
ρ_G = 0.9706	0.1073	0.8636	0.0343	0.2442
σ_A = 0.0100	0.1792	2.0089	0.0784	0.3448
σ_ν = 0.0282	0.1124	0.8746	0.0349	0.2842
σ_Z = 0.0468	0.1260	0.8929	0.0358	0.3091
σ_G = 0.0113	0.1070	0.8643	0.0344	0.2442

Notes: The table contains the compensating variations of three different targeting rules as well as a Taylor rule in the medium scale model when each parameter is set at the 10th percentile of its post distribution holding all other parameters fixed at the mode of the posterior.